University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2018**

Problem Set #1

(due 10/9/18)

1. Consider an economy with three local communities of equal population size. Individuals are identical within each community *i*, with preferences over private goods *c* and local public goods *g* governed by the utility function $log(c - a_i) + log(g)$ and endowment y_i of the private good. For simplicity, assume also that each community has one individual. Both private and public goods have a price of 1 and public goods must be purchased separately for each jurisdiction (i.e., there are no spillovers in public goods consumption across jurisdictions).

   A. Suppose first that each community chooses its own level of spending on the public good. Solve for the level of public spending and the level of utility in each community as a function of the parameters a_i and y_i.

   B. Now, suppose that all public spending is centrally financed by a proportional income tax at rate *t*, where *t* is the same across communities and each community's level of the public good equals one-third of revenue raised in the whole economy. Thus, once the level of *t* is determined, private and public goods consumption in each community are also determined. Assume that *t* is chosen by a simple majority vote.

      i. Solve for each community's preferred level of *t* and show that preferences over *t* are single-peaked in each community, so that the level of *t* chosen by majority vote will be that of the median voter. What will determine who the median voter is?

      ii. Under what condition will total public spending be greater under central provision than under local provision (case A.)?

      iii. Is the equilibrium Pareto-optimal under central provision?

   C. Now, suppose that before voting on the level of public goods, individuals vote on whether to use local provision or central provision. Also suppose that $\bar{y} < y_1$, where $\bar{y} = (y_1 + y_2 + y_3)/3$ is average community income. Show that community 1 will vote for local provision. (*Hint*: First compare the outcomes in the case for which community 1 is the median voter under central provision.)

2. Consider an economy with fixed producer prices and a representative household that maximizes utility, which is a function of one consumption good and two types of leisure, perhaps the leisure of two spouses,

$$U(c, l_1, l_2) = c^{\alpha_0} l_1^{\alpha_1} l_2^{\alpha_2}$$

   (where $\alpha_0+\alpha_1+\alpha_2 = 1$), subject to the budget constraint:

$$pc = w_1(1 - l_1) + w_2(1 - l_2)$$

A. In order to set the problem up in terms of transactions between households and firms, rewrite the household's utility function and budget constraint in terms of consumption and labor supply, $L_1 = (1-l_1)$ and $L_2 = (1-l_2)$.

B. Derive expressions for the compensated demand for c and compensated supplies of L_1 and L_2.

C. Suppose that the government wishes to raise a fixed amount of revenue from the household using separate proportional taxes on L_1 and L_2 Based on the standard three-good analysis, use your answers to part B to derive a condition in terms of exogenous variables – wage rates and utility function parameters – for uniform taxation to be optimal.

3. Suppose that a market consists of a large number of producers with the production function $x = aL^{\gamma}E^{1-\gamma}$, where L is a "clean" input, labor, and E is a "dirty" input, energy, that generates emissions in fixed proportion to energy use. The fixed market prices for labor and energy are w and q, respectively, and the price for x determined by supply and demand in this market is p. Assume that the market demand curve satisfies $x = bp^{-\varepsilon}$.

   A. The government wishes to use a Pigouvian tax on energy to reduce energy use to a fraction θ of its original level. By how much must the tax increase the price of energy? (*Hint*: First solve for energy use as a function of factor prices in equilibrium, as determined by the demand and producer cost functions.)

   B. At this new equilibrium, what fraction of the decline in energy use is attributable to the decline in output, holding factor proportions fixed? How much is due to the shift in factor proportions, holding output at its reduced level?

   C. Now, suppose that the government attacks the problem by issuing tradable permits instead of imposing a tax, with the number of permits issued equal to target energy use.

      i. What will the equilibrium price of the permits be, relative to the initial price of energy?

      ii. What share of the permits would the government need to provide, free of charge, to existing producers in order to allow them to maintain their original level of profits?